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SIMULATION OF THE STRUCTURE AND CALCULATION OF THE THERMAL CONDUCTIVITY OF NAPPED COMPOSITES

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We propose a model of the structure of a napped composite. Characteristic trends in the structure of the material are delineated, and the effective thermal conductivity of the model structure is calculated for these trends with allowance for conduction and radiation.

Object of Investigation and Statement of the Problem. A high-temperature insulation with a nap structure ("vorsolin") made of refractory fibers is distinguished by considerable resistance to pulsed temperature effects, accelerations, and vibrations.

Usually, the time allotted for studying and selecting the variants of the structure of a heatproof layer is very limited, and the cost of the components and technological expenditures are high. Under these conditions a tendency has developed in recent years toward a complex theoretical and experimental investigation of the properties of composites. A large number of variants of a material with previously specified thermophysical properties are calculated as early as at the stage of computer-aided design, and only those offering the greatest promise (judging by the set of thermophysical, technological, and operation characteristics) are selected for experimental verification.

A typical structure of a heatproof nap layer (vorsolin) is depicted in Fig. 1. On base 1 (fabric made from refractory fibers) adjacent to a protected surface, fastening refractory threads 2 fix refractory nap loops 3 of the main heatproof layer. Depending on the intensity and duration of the temperature effect on the heat insulation, the temperature difference over the thickness of the nap heat shielding may constitute from tens to thousands of degrees (a fused layer 4 consisting of the material of the fibers can be seen on the surface of the vorsolin).

Simulation of the Vorsolin Structure. To simplify the subsequent mathematical analysis, following the recommendations given in [1], we divide the napped material by conventional isothermal planes into several layers located sequentially with respect to the heat flux affecting the protected surface. Within the limits of each layer the thermophysical properties of the components of the nap layer are assumed to be known for the mean temperature of the layer. The outer layer with the maximum temperature will be called the layer of a "melt." If the prescribed temperature of the outer surface is lower than the melting temperature of the fibers of the threads, the layer of the melt is absent (its thickness is assumed to be equal to zero).

Next, we distinguish in the material (Fig. 1b) the layers of "loop arcs" and "nap sticks." If the outside loops are cut, the layer of loop arcs is absent. Then comes the layer of "fastening threads." The lower layer is the layer of the "fabric," i.e., of the base of the napped material immediately adjacent to the surface being protected.

The specific (per unit area of the protected surface) thermal resistance of the vorsolin in the $X-Y$ plane is composed of the specific thermal resistances of the layers distinguished above. We will present the main stages in the calculation of the specific thermal resistance and the effective thermal conductivity of the napped heat shielding.

Preliminary estimates made according to the recommendations of [1–4] show that the basic mechanisms involved in the transfer of heat through the heat shielding layer is conduction through fibers and molecular and radiant heat transfer in the pores between the fibers.

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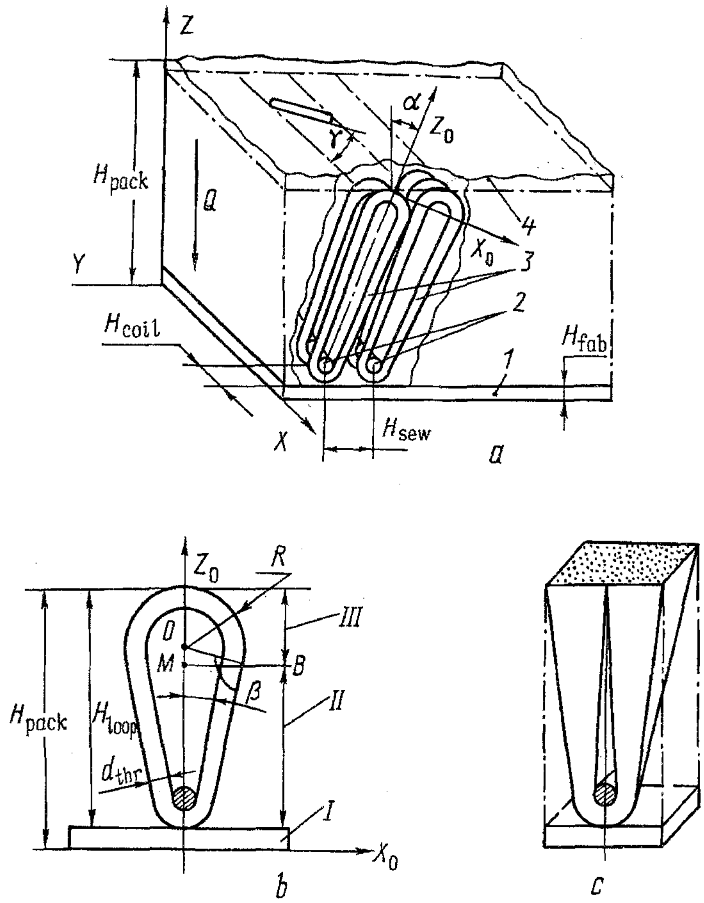

Fig. 1. Schematic representation of a fragment of the structure of a heatproof napped composite material (vorsolin): a – 1) base attached to the protected surface by a heat-resistant adhesive or by fastening threads; 2, 3) loops of nap from thermostable threads; 4) a layer of melt (when $T_{oper} > T_{melt}$); b – uncut loop of vorsolin: *I*) base (fabric); *II*) layer of nap sticks; *III*) layer of loop arcs; c – cut loop with fluffed thread.

A distinctive attribute of the vorsolin structure making it difficult to use the already familiar models of fibrous materials and techniques of calculation of their thermophysical characteristics is the inclined and "twisted" orientation of nap loops with respect to the heat flux direction. To take into account the inclination and twisting of the nap loops, the following procedure is used [1]. Let us consider a structurally anisotropic material with the thermal conductivities λ_z and λ_x in the directions Z and X. Then, the effective thermal conductivity of the material λ_φ in the direction n, making the angle φ with the Z axis, is determined by the relation [1]

$$\lambda_\varphi = \lambda_z \cos^2 \varphi + \lambda_x \sin^2 \varphi . \tag{1}$$

We shall present the sequence of calculation of the effective thermal conductivity of vorsolin, which is performed in several stages.

At the first stage we calculate the effective thermal conductivity of the threads of the nap and the "fabric base" along, $\lambda_{||}$, and across $\lambda_\perp$, the fibers with account for heat transfer by conduction through the fibers and the gas phase and by radiation.

A "local" coordinate system is introduced for each of the material layers distinguished above. The Z_0 axis (longitudinal) coincides with the direction "characteristic" for a given layer. For example, for the layer of loop arcs the Z_0 axis lies in the plane of a loop and divides it into two symmetric parts (Fig. 1). The X_0 and Y_0 axes are orthogonal to the Z_0 axis.

At the second stage we calculate the conductive and radiative components of the effective thermal conductivity of the distinguished layers in the direction of the axes of the local coordinate system X_0, Y_0, Z_0.

At the third stage we determine the specific thermal resistance of each layer in the directions of the main coordinate system X, Y, Z and the effective characteristics of the vorsolin structure as a whole.

Let us consider in more detail the above-enumerated stages of calculation.

Effective Thermal Conductivity of the Threads of the Nap and the Fabric. The threads have a porous structure that is twisted from elementary fibers made of materials capable of operating for the required time at specific temperatures. Heat is transferred through the thread by conduction and radiation. The effective thermal conductivities of the thread $\lambda_{||}$ and $\lambda_{\perp}$ are determined from the simplest relations [5]:

$$\lambda_{\parallel} = \lambda_f (1 - \Pi_{thr}) + \lambda_g \Pi_{thr} , \quad (2)$$

$$\lambda_{\perp} = \lambda_g (1 - \sqrt{1 - \Pi_{thr}} + \left(\frac{1}{\lambda_f} + \frac{1 - \sqrt{1 - \Pi_{thr}}}{\lambda_g \sqrt{1 - \Pi_{thr}}} \right) , \quad (3)$$

$$\lambda_g = \lambda_{mol} + \lambda_{rad} . \quad (4)$$

The molecular component of the thermal conductivity of the gas [1] is

$$\lambda_{mol} = \lambda_{g0}/(1 + B/(p\bar{\delta})) , \quad \bar{\delta} = (\pi/4) (d_f/(1 - \Pi_{thr})) , \quad B = 1.75 \cdot 10^{-4} \ \text{Pa} \cdot \text{m} ; \quad (5)$$

and the radiant component [6] is

$$\begin{gathered} \lambda_{rad} = (16/3) \, n_{thr}^2 \sigma \overline{T}_i^3 \, Y (\varepsilon_w , \tau_{opt})/\alpha_{att} , \\ n_{thr} = 1 + (n_f - 1) (1 - \Pi_{thr}) , \quad \tau_{opt} = \alpha_{att} \, l_i . \end{gathered} \quad (6)$$

Let us assume that $l_i = d_{thr}$ when the thermal conductivity of the thread is calculated across it and that $l_i = 100 d_{thr}$ when the thermal conductivity is calculated along the thread.

The coefficient of attenuation of radiation can be determined for two variants. If the attenuation of radiation occurs mainly at the expense of its absorption on the surface of the fibers, then [1]

$$\alpha_{att} = 4 (1 - \Pi_{thr})^2 (2 - \varepsilon_f)/(\pi d_f) , \quad (7)$$

and if it occurs mainly at the expense of its dissipation on the surface of the fibers, then

$$\alpha_{att} = \frac{8}{3\pi d_f} \frac{\rho}{\rho_f} \Gamma_{sc} . \quad (8)$$

In this case for "thick" ($X_{arc} > 1$) fibers

$$\begin{gathered} \Gamma_{sc} = 2 - (4/X_{arc}) \sin X_{arc} + (4/X_{arc}^2) (1 - \cos X_{arc}) , \\ X_{arc} = \frac{2\pi d_f (n_f - 1)/T_i}{2900} \end{gathered} \quad (9)$$

(d_f is in μm).

When radiation is scattered on thin, $X_{arc} < 0.6/n$, fibers (Rayleigh scattering)

$$\Gamma_{sc} = (8/3) \, X_{arc}^4 \, ((n_f^2 - 1)/(n_f^2 + 1))^2 . \quad (10)$$

Relations (2)-(10) determine the effective thermal conductivity of fibers at the mean temperatures $\overline{T}_i$ of each of the distinguished layers.

Elementary Cells of the Layers and Local Coordinate Systems in the Layers. Layer of Loop Arcs. An elementary cell of the layer of loop arcs represents a parallelepiped with the dimensions of the base $1 \times H_{sew}$ (in the X, Y plane) and with the length of the third side determined so as to include the curved portion of the loop arc. The angle between this side and the Z axis of the general coordinate system will be denoted by α:

$$\alpha = \arccos \frac{H_{pack} - H_{fab}}{H_{loop}} . \tag{11}$$

We can show that the length of the inclined side in the elementary cell of the layer of loop arcs is equal to $R(1 + \sin \beta)$, where

$$\beta = \arcsin \frac{R}{H_{loop} - R} . \tag{12}$$

All the loops in a row are inclined to the line of sewing (parallel to the X axis) at angle γ. We assume that the loops of neighboring rows touch one another (there are no through gaps between rows of loops in the plane of a loop). The value of the angle γ is determined by the relation

$$\gamma = \arcsin (H_{sew}/(2R)) . \tag{13}$$

The distance between neighboring loops in a row along the X axis is determined by the step H_{coil} with which the loops are coiled round the sewing thread.

Let us introduce the local coordinate system X_0, Y_0, Z_0, which is obtained from the general system X, Y, Z by rotating the Z, Y plane about the X axis through the angle α in the clockwise direction and then the X_0, Y_0 plane about the Z_0 axis through the angle γ in the anticlockwise direction. In this coordinate system the loops will lie in planes parallel to the X_0, Z_0 plane.

To calculate the thermal conductivity of the layer of arcs, we use the simplified model of its structure shown in Fig. 2a and b. In this model a thread is replaced by a bar of equal area of square cross section with the side a_{bar}:

$$a_{bar} = (1/2)\, d_{thr} \sqrt{\pi} . \tag{14}$$

The dimension H_1 will be taken equal to the length of the inclined side of an elementary cell of the layer of arcs (see above):

$$H_1 = R\,(1 + \sin \beta) , \tag{15}$$

and we determine L from the condition of equality between the volumes of the arc of the thread and the rectangular bar simulating it:

$$L = R\,(\pi/2 + \beta) - H_1 + a_{bar} , \tag{16}$$

and a_{por} from the condition of equality between the porosity of the model and the real material

$$a_{por} = \frac{a_{bar}\,(a_{bar}\,(H_1 + L) - a_{bar}^2 - LH_1\,(1 - \Pi_{arc}))}{LH_1\,(1 - \Pi_{arc})} . \tag{17}$$

We can show that

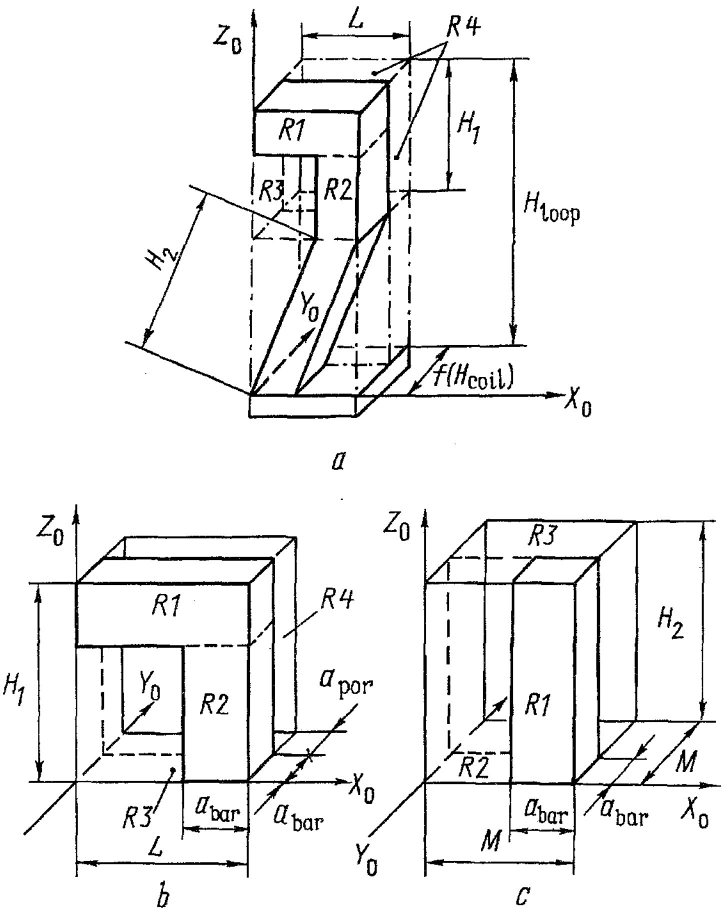

Fig. 2. Simplified model of a nap layer (a), elementary cells of the layer of loop arcs (b) and of the layer of nap sticks (c).

$$\Pi_{\mathrm{arc}} = 1 - \frac{\pi d_{\mathrm{thr}}^2 (\pi + 2\beta)}{4 H_{\mathrm{sew}} H_{\mathrm{coil}} R (1 + \sin\beta) \cos\alpha} . \tag{18}$$

Thus, expressions (11)-(18) determine all the geometric parameters of the simplified model of the structure of the layer of arcs.

Layer of Nap Sticks. An elementary cell of the layer of nap sticks also represents a parallelepiped. The length of an inclined side of the parallelepiped is determined proceeding from the fact that only linear portions of threads could fall into a given cell. We can show that the length of an inclined side is $H_2 = R \cos^2\beta/\sin\beta$. In the simplified model of the layer (Fig. 2a and c) the fibers are oriented vertically.

The local coordinate system differs from the local coordinate system of the layer of loop arcs by an additional rotation about the Y_0 axis through the angle β. The height H_2 of the model is equal to the length of the linear portion of the thread:

$$H_2 = R/\tan\beta . \tag{19}$$

The dimension a_{bar} is given by expression (14), and the value of M can be found from the condition of equality between the porosities of the nap sticks and the model:

$$M = \frac{a_{\mathrm{bar}}}{\sqrt{1 - \Pi_{\mathrm{n.st}}}} . \tag{20}$$

We can show that $\Pi_{\mathrm{n.st}}$ (the porosity of the layer of nap sticks) is determined from the expression

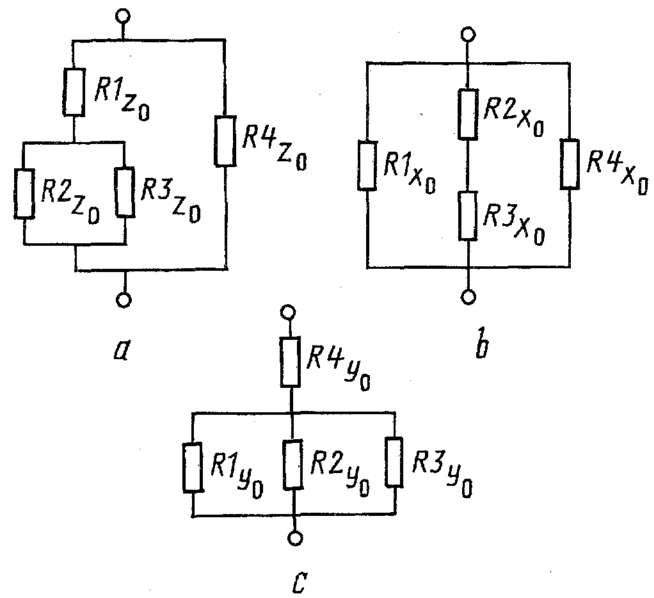

Fig. 3. Schematic of combining the thermal resistances of individual parts.

$$\Pi_{\text{n.st}} = 1 - \frac{\pi d_{\text{thr}}^2}{2H_{\text{coil}} H_{\text{sew}} \cos\beta \cos\alpha} . \tag{21}$$

Thus, relations (11)-(14) and (19)-(21) determine all the geometric parameters of the model of the layer of nap sticks.

Layer of Fastening of Nap Sticks. Taking into account the fact that the layer of fastening occupies a small fraction of the volume of the material, we consider that the effect of this layer on the effective thermal conductivity of the heatproof packet can be taken into account by increasing the thickness of the fabric layer by the value d_{thr}.

Fabric Layer. The means of calculating the effective thermal conductivity of fabrics are detailed in [1], and therefore we will not consider it here to save space.

Calculation of the Effective Thermal Conductivity of Simplified Models. Let us consider the basic layers of loop arcs and nap sticks, since the calculation of the thermal conductivities of the remaining layers does not involve any difficulties.

Let us mentally divide the model of the arcs by a system of isothermal and adiabatic planes, as shown in Fig. 2a and b.

The regions with the thermal resistance $R1$ correspond to the upper portion of the arc of the heat-shielding thread, $R2$ to the lower portion of the arc, R_3 to the pore region bounded by the arc, $R4$ to the through pores. The scheme of combining the thermal resistances for calculating the effective thermal conductivity in the direction Z_0 is shown in Fig. 3.

The values of the thermal resistances are determined from the formulas

$$R1_{z_0} = 1/(\lambda_\perp L) , \quad R2_{z_0} = (H_1 - a_{\text{bar}})/(\lambda_{\parallel} a_{\text{bar}}^2) ,$$
$$R3_{z_0} = \frac{H_1 - a_{\text{bar}}}{\lambda_g (L - a_{\text{bar}}) a_{\text{bar}}} , \quad R4_{z_0} = H_1/(\lambda_g L a_{\text{por}}) . \tag{22}$$

All of the geometric parameters in Eqs. (22) are known; $\lambda_{||}$ and $\lambda_\perp$ are determined by relations (2) and (3). It is necessary to determine the thermal conductivity of the gas filling the space between the threads of the heat shielding.

Let us avail ourselves of the approach given above. The thermal conductivity of the gas in pores λ_g is composed of molecular and radiant components (4). The molecular component is calculated from formula (5). The parameters λ_g, p, and B are assumed to be specified, while the mean free path of the molecules $\bar{\delta}$ in the direction Z_0 is taken to be equal to half the loop height in the direction Z_0:

$$\bar{\delta} = H_{\text{loop}}/2 . \tag{23}$$

We calculate the radiant component from formula (6). The effective refractive index of the layer of arcs with allowance for its porosity is determined by an expression similar to relation (10):

$$n_{\text{arc}} = 1 + (n_{\text{thr}} - 1)(1 - \Pi_{\text{arc}}) , \tag{24}$$

The coefficient of attenuation of radiation can be calculated from formula (7). In the given case the area of an elementary cell that absorbs radiation in the direction Z_0 is equal to $a_{\text{bar}}L$, while teh total area of the elementary cell in the direction normal to Z_0 is equal to $L(a_{\text{bar}} + a_{\text{por}})$. Taking into account relation (23), we obtain

$$\alpha_{\text{att}} = \frac{2(2-\varepsilon)\, a_{\text{bar}}}{H_{\text{loop}}(a_{\text{bar}} + a_{\text{por}})} . \tag{25}$$

The value of l entering into Eq. (6) is taken to be equal to $H_{\text{pack}}/\cos\alpha$, since the layer of the nap has precisely this size in the direction Z_0.

Thus, using the above relations, it is possible to calculate all the quantities entering into formulas (22) and, consequently, to determine the effective thermal conductivity of the layer of arcs in the direction Z_0.

The calculation of the effective thermal conductivity of the layer of arcs in the directions X_0 and Y_0 and of the effective thermal conductivity of the layer of nap threads for all three directions of the axes in the local coordinate system is carried out in the same way.

Calculation of the Effective Thermal Conductivities of Layers along the Axes of the General Coordinate System. Suppose the thermal conductivity of the layer of arcs in the direction of the axes of the local coordinate system of this layer has been determined by the above-described technique and has the values λ_{x0}, λ_{y0}, and λ_{z0}. To determine the themal conductivity of this layer in the direction of the axes of the general system of coordinates, we use relations similar to relation (1).

Let us recall that the system of coordinates X_0, Y_0, Z_0 was obtained from X, Y, Z by two successive rotations: about the X axis through the angle α and about the Z_0 axis through the angle γ. For the angle γ we can write

$$\begin{aligned} \lambda_{x_1} &= \lambda_{x_0}\cos^2\gamma + \lambda_{y_0}\sin^2\gamma , \\ \lambda_{y_1} &= \lambda_{y_0}\cos^2\gamma + \lambda_{x_0}\sin^2\gamma , \\ \lambda_{z_1} &= \lambda_{z_0} , \end{aligned} \tag{26}$$

and for the angle α

$$\begin{aligned} \lambda_x &= \lambda_{x_1} , \\ \lambda_y &= \lambda_{y_1}\cos^2\alpha + \lambda_{z_1}\sin^2\alpha , \\ \lambda_z &= \lambda_{z_1}\cos^2\alpha + \lambda_{y_1}\sin^2\alpha . \end{aligned} \tag{27}$$

Similarly we determine the values of the effective thermal conductivity for the layer of nap sticks.

The procedure described in this work was implemented in the form of a computer program, which makes it possible at the design stage to calculate the effective thermal conductivity of a fibrous anisotropic material having a complex structure and exposed to large temperature differences over its thickness. The initial data for the program are the geometric characteristics of the material, the temperature dependences of the thermal conductivity of the material components, and the temperatures at the boundaries of the heat-shielding packet. As a result of calculation we determined the effective thermal conductivities of each of the layers and of the entire material in the directions of the axes of the general coordinate system.

NOTATION

$\alpha, \beta, \gamma, \varphi$, angles; $\lambda_x, \lambda_y, \lambda_z, \lambda_\varphi$, thermal conductivities of the material in the direction of the X, Y, Z axes of the "global" coordinate system and in direction φ; $\lambda_{x0}, \lambda_{y0}, \lambda_{z0}, \lambda_{x1}, \lambda_{y1}, \lambda_{z1}$, thermal conductivities of the material in the direction of the axes of the "local" coordinate system; $\lambda_{||}$, $\lambda_{\perp}$, λ_f, thermal conductivity of a thread along and across the fibers and of a fiber proper; λ_g, λ_{g0}, λ_{mol}, λ_{rad}, thermal conductivities of the gas in the pores of a thread at pressures p and p_0, respectively, molecular and radiant components of the thermal conductivity of the gas component in the pores; Π_{thr}, Π_{arc}, $\Pi_{n.st}$, porosities of a thread, layers of arcs, and nap sticks; B, molecular constant; $\overline{\delta}$, mean free path of gas molecules in the pores of a thread; d_{thr}, d_f, diameters of a thread and a fiber from which the thread is composed; $\overline{T}_i$, mean temperatyure of the i-th layer; l_i, τ_{opt}, geometrical and optical thicknesses of a layer in which radiation attenuation takes place; a_{att}, coefficient of radiation attenuation; σ, Stefan-Boltzmann constant; ε_w, ε_f, emissivities of the walls and the material of the fibers; $Y(\varepsilon_w, \tau_{opt})$, Poltz function; n_f, n_{thr}, n_{arc}, refractive indices of a fiber, a thread, and the layer of arcs; ρ, ρ_f, density of a thread and the material of a fiber; H_{pack}, H_{fab}, H_{loop}, H_{sew}, H_{coil}, R, geometric parameters of the real material; a_{bar}, a_{por}, H_1, H_2, L, M, geometric parameters of the model structure; $R1$, $R2$, $R3$, $R4$, thermal resistances.

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